The Munich Lectures in Economics 2011

Time and the Generations

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Lecture 2

"Time, Self, and the Generations"

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Readings:

P. Dasgupta (2001), *Human Well-Being and the Natural Environment* (Oxford: Oxford University Press). 2nd Ed. (2004).

P. Dasgupta (2008), "Discounting Climate Change," *Journal of Risk and Uncertainty*, 37, 141-169.

In evaluating an economy, there are five questions we can ask:
(A) How is the economy doing?
(B) How has it performed in recent years?
(C) How is it likely to perform under "business as usual"?
(D) How is it likely to perform under alternative policies?
(E) What policies should be pursued there?

National income accounts offer information relevant for answering (A), although it does so in an unsatisfactory way. Policy evaluation, including *project evaluation*, is a way to answer questions (D) and (E). The idea is to evaluate an economy *at a point in time* before and after a hypothetical perturbation has been made to it. In contrast, the literature on "sustainable development" answers questions (B) and (C) by evaluating economic change when the perturbation is *the passage of time* itself.

We discuss the two types of evaluation sequentially.

Question (A) stands apart from questions (B) to (E), at least if conventional practice among national income statisticians is any guide. For it is common practice to summarize the state of an economy by its GDP, or equivalently its (gross) domestic income.

Why GDP is inadequate for answering (B)-(E)?

Population is taken to be constant (say, 1). Assume time is dicrete: $t = 0,1,2,...$ If C is consumption, $U(C)$ is social well-being. Let $C(t)$ denote consumption at t. A consumption forecast is the indefinite sequence $\{C(0),C(1),...,C(t),...\}$.

Generalized Utilitarianism:

Let $V(t)$ be intergenerational well-being at t. Intergenerational well-being at $t = 0$ is assumed to be:

$$V(0) = {}_0\Sigma[U(C(t))/(1+\delta)^t], \qquad \delta \geq 0. \qquad (1)$$

U is unique up to positive affine transformations. Δ is the time discount rate.

How should the social evaluator identify U?

(1) Infer U from the choices people make as they go about their lives ("revealed preference"). (2) Choice behind the veil of ignorance (Harsanyi-Rawls). (3) Altruistic parents concerned with dynastic welfare. (4) V is a numerical representation of a set of ethical requirements on orderings over consumption sequences (Koopmans). (5) Philosophical introspection (Mill-Ramsey)!

Philosophical Intuitionism

Let Ω be a set of bounded infinite sequences $\{C(0), C(1), ..., C(t), ...\}$. If U (a bounded function of C), is well-being, Koopmans' (1960) axioms on Ω imply

$$W_0 = {}_{t=0}\Sigma^{\infty}\{G[U(t))]/(1+\delta)^t\} = {}_{t=0}\Sigma^{\infty}\{G[U(C(t))]/(1+\delta)^t\}, \qquad \delta > 0.$$

G is an increasing, bounded function of U. The axioms specify neither δ nor the form of G.

[The latter is true only if the measurement of U does not afford the social evaluator any degree of freedom (i.e., U is unique up to the identity transformation), which is what Koopmans assumed in his work. Restrictions on G appear if, say, U is unique up to positive linear transformations (i.e., proportional transformations). But no matter what restrictions are entailed on the form of G, the function is unique up to positive affine transformations.]

References:

Koopmans, T.C. (1960), "Stationary Ordinal Utility and Impatience," *Econometrica*, 28(2), 287-309.

Diamond, P.A. (1965), "The Evaluation of Infinite Utility Streams," *Econometrica*, 33(1), 170-77.

Central case:

$U(C) = C^{(1-\eta)}/(1-\eta)$, for $\eta > 0$ and $\eta \neq 1$,

and $U(C) = \ln C$, corresponding to $\eta = 1$. (2)

NB: The larger is η, the greater is the curvature of $U(C)$. $U(C)$ is bounded above but unbounded below if $\eta > 1$, but $U(C)$ is bounded below but unbounded above if $\eta < 1$. η is the elastcity of marginal felicity. We show below that it is a measure of *inequality* (and *risk*; see below) aversion.

Let $g(C(t))$ be the % rate of change in $C(t)$ along the forecast. Use (2) in (1) to obtain an expression for ρ. Show that

$$1 + \rho_t = (1+\delta)(1+g(C(t)))^{\eta}. \tag{3}$$

Suppose δ and $g(C(t))$ are both small. Then (3) becomes

$$\rho(t) = \delta + \eta g(C(t)). \tag{4}$$

If the interval between dates was to be made smaller and smaller, (4) would be a better and better approximation. (If time is continuous, (4) is an equality.)

δ, η, and the forecast, $g(C(t))$, together determine $\rho(t)$. Observe that $\rho(t)$ increases with δ and $g(C(t))$, respectively, and increases with η *if and only if* $g(C(t)) > 0$. So (3) (and (4)) reflects reasons (A) and (B) for the sign of ρ. δ is "impatience" and η is the index of inequality aversion.

Proposition 1. η *is the index of the aversion society ought to display toward consumption inequality among people - be they people in the same period or in different periods*.

(3) says that when $g(C(t)) > 0$, δ and η play similar roles in the determination of $\rho(t)$: a higher value of either parameter would reflect a greater aversion toward consumption inequality. Which may explain why it hasn't been uncommon to suppose that higher values of δ reflect a greater ethical concern for consumption equality. But if $g(C(t)) < 0$, δ and η assume diametrically opposite features: in contrast to η, higher values of δ raise $\rho(t)$, implying an ethical preference for even greater inequality in consumption across the generations.

Observations on equation (4):

(a) $\rho(t)$ is not a primary ethical object, it has to be derived from an overall conception of intergenerational well-being and the consumption forecast: consumption discount rates cannot be plucked from air.

(b) Just as growing consumption provides a reason why discount rates in use in social cost-benefit analysis should be positive, declining consumption would be a reason why they could be negative. Example: Suppose $\delta = 0$, $\eta = 2$, and $g(C(t))$ = -1% per year. Then $\rho(t)$ = -2% per year. Such reasoning assumes importance when we come to discuss that people in the tropics, who are in any case very poor, will very likely suffer greatly from climate change under business as usual. The reasoning takes on an interesting application when we come to consider uncertainty in future consumption.

(c) If intertemporal external diseconomies are substantial, as is the case with climate change under business as usual, both $\rho(t)$ and private rates of return on investment could be positive for a period of time, even while the social rate of return on investment is negative.

(d) Only in a fully optimizing economy is it appropriate to discount future consumption costs and benefits at the rate that reflects the direct opportunity cost of capital. In imperfect economies $\rho(t)$ should be used to discount consumption costs and benefits, but the capital deployed in projects ought to be revalued so as to take account of the differences between $\rho(t)$ and the various rates of return on investment.

(e) Unless consumption is forecast to remain constant, social discount rates depend on the numeraire: $\rho(t) = \delta$ if and only if $g(C(t)) = 0$. (f) If $g(C(t))$ varies with time, so does $\rho(t)$. For example, suppose it is forecast that long-run consumption growth is not sustainable but will decline at a constant rate of 1% a year - from the current figure of 2% a year to zero. Suppose $\delta = 0$ and $\eta = 2$. In that case $\rho(t)$ will decline over time at 1% a year, from a current-high 4% a year, to zero.

Examples from the Economics of Climate Change:

Cline (1992): $\delta = 0$; $\eta = 1.5$

Nordhaus (1994): $\delta = 3\%$ a year; $\eta = 1$

Stern (2006): $\delta = 0.1\%$ a year; $\eta = 1$

NB: In the context of (4), the authors are close in their choice of η. Notice also how close Cline and Stern are in their specifications of δ.

The point estimate of consumption growth under business as usual in Stern (2006) is $g(C(t)) = 1.3\%$ a year. Use this in equation (4) to obtain:

$\rho(t) = 2.05\%$ a year for Cline

$\rho(t) = 4.30\%$ a year for Nordhaus

$\rho(t) = 1.40\%$ a year for Stern

That is why Cline and Stern arrive at similar conclusions and why they differ in their recommendation from Nordhaus.

The Fully Optimum Economy

Suppose $\eta \geq 1$. Let $K(t)$ be the economy's *wealth* at t and let the economy's accumulation process be

$$K(t+1) = [K(t)-C(t)](1+r), \quad K(0)\ (> 0) \text{ is given.} \tag{5}$$

Assume $r > \delta$. In a fully optimum economy, the $\{C(t)\}$ that society chooses maximizes expression (1), subject to the accumulation equation (5). NB: Under our hypotheses an optimum exists and satisfies the condition:

$$\rho(t) = r, \qquad \text{all } t \geq 0. \tag{6}$$

It is only in a fully optimum economy that the direct opportunity cost of capital should be used for discounting future benefits and costs.

What does an optimum $\{C(t)\}$ look like? Using (5) and (6), it can be shown that $C(t)$ grows at the compound rate, g, where

$$C(t+1)/C(t) - 1 = g = [(1+r)/(1+\delta)]^{1/\eta} - 1. \qquad (7)$$

If r and δ are small, then g is small, and (7) becomes the approximation

$$g = (r-\delta)/\eta. \qquad (8)$$

(Equation (8) is exact in continuous time.)

Let the optimum saving rate, $[K(t)-C(t)]/K(t)$, be s. Then

$$s = (1+r)^{-(\eta-1)/\eta}(1+\delta)^{-1/\eta}. \tag{9}$$

Proposition 2. *The optimum saving rate is a decreasing function of* η *and* δ. *If, holding* δ *and r constant, larger and larger values of* η *are admitted, s declines to* $(1+r)^{-1}$.

NB: The "Rawlsian" case is $\eta = \infty$.

Note: Net saving is zero if $s = 1/(1+r)$. Normalise round that figure. Moreover, the maximum possible rate of saving is 1, implying that the range of non-negative saving rates is $[(1+r)^{-1}, 1]$. Since the saving-wealth ratio is $[K(t)-C(t)]/K(t)$, its normalised value is $[(K(t)-C(t)]/K(t)-(1+r)^{-1}]/[1-(1+r)^{-1}]$. Now, output at t+1 is $rK(t)$. Confirm that the normalised saving-wealth ratio is none other than the more familiar saving-output ratio.

Let s^* be the optimum saving-output ratio. If r and δ are both small, then (9) becomes

$$\tilde{s}^* = (r-\delta)/\eta r. \tag{10}$$

Example: Let r = 4% a year. (10) says that at δ = 0.1% a year, the optimum saving-output ratio is 97%. This is an absurdly high rate of saving out of income, suggesting that η = 1 (the log case) is misleading.

Uncertain Production Economy

Suppose at each date, $\ln(1+r)$ in equation (5) is distributed independently, identically, and normally, with mean μ and variance σ^2. Let $\bar{r}$ be the expected value of $\tilde{r}$. Assume $\bar{r} > \delta$. Obviously, $\bar{r}$ is a function of μ and σ; as is the variance of $\tilde{r}$. Assume that $\eta \geq 1$. Let s^{**} be the optimum saving-output ratio and $\bar{r}$ and δ are both small. Then

$$\tilde{s}^{**} = (\bar{r}-\delta)/\eta\bar{r} + (\eta-1)\sigma^2/2\bar{r}. \quad (11)$$

Proposition 3. η *is not only an index of inequality aversion, it is also an index of risk aversion. At the saving rate* s^{**}*, future generations can be expected to be richer than the present generation. Because of the growth effect, larger values of* η *recommend earlier generations to save less for the future (the equity motive). However, as future productivity is uncertain, larger values of* η *recommend earlier generations to save more (the precautionary motive). The combined effect depends on the parameters* η, δ, $\bar{r}$, *and* σ.

Large Uncertainties: Equation (11) says that $s^{**} \geq 1$ if

$$\sigma^2/2 \geq \ln(1+\delta)/\eta(\eta-1) + \ln(1+\overline{r})/\eta. \tag{12}$$

As $s^{**} \geq 1$ is nonsensical, we can summarise the finding as

Proposition 4. *If* σ *satisfies inequality (12), no optimum policy exists*.

Discuss ways out of Proposition 4.

Ramsey-Koopmans-Harsanyi Formulation: Fundemental Weakness

If the unit of time in expression (1) is taken to be a generation's span, a person's life remains a black box. On the other hand, to assume the unit of time to be less than a generation's span raises a deep conceptual problem:

It does not acknowledge *personhood*.

Consider expression (1). Interpret *U*(t) to be the sum of the utilities of all who are alive at t. Expression (1) says that not only is someone's lifetime welfare the discounted sum of her utilities, the rate used for discounting those utilites is also the rate that should be used to discount future people's *lifetime* welfares.

To put it another way, expression (1) says that the rate of substitution between an individual's utilities in two periods of time is the same as the rate of substitution between the utilities of two individuals in those same two periods of time. Such an ethical move can be justified only if it is presumed that a person is an entirely separate self in each period of her life. The ethics embodying expression (1) only values the flow of utilities, it doesn't acknowledge the significance of the bearers of those utilities.

One can argue that the choices a person makes so as to give shape to her life is a personal matter, something over which no meddling ethicist should have a say. "How much should I save for my children?" involves considerations that differ from the reasoning a person engages in when asking, "How should I spread out *my* consumption over time?" The first question involves a combination of affection and obligation (a person's treatment of her children), whereas the latter involves an ethics that shapes our conception of an integrated life. Expression (1) addresses the former question, but adopts an unsatisfactory position regarding the latter. The formula doesn't accommodate the concept of the *self*, living through time.

Dasgupta-Maskin:

Consider first a single dynasty. Assume generations don't overlap. To simplify the notation, let t be continuous. A person lives for T years and is replaced by a single descendent at her death. We begin by considering a single dynasty.[1] Let δ (> 0) be the rate of impatience people display toward their own utility. Denote by $C(t)$ consumption at t (≥ 0) of that member of the dynasty who will be alive then. If $U(C(t))$ denotes her utility at t, the lifetime welfare of someone born at iT ($i = 0,1,...$), will be

$$V_{iT} = \int_{iT}^{(i+1)T} \{U(C(t))\} e^{-\delta(t-iT)} dt. \quad (13)$$

[1] The demographic structure I assume here is the same as the one in Meade (1966). But the normative structure in his paper was very different from the one Maskin and I are trying to capture in our joint work.

Next consider someone born at $t = 0$. Call her person-0. Using expression (13), her dynasty's well-being is:

$$W_0 = \sum_{i=0}^{\infty}[V_{iT}] = \sum_{i=0}^{\infty}\left[\int_{iT}^{(i+1)T}\{U(C(t))\}\mathrm{e}^{-\delta(t-iT)}\right]\mathrm{d}t.^2 \quad (14)$$

Expression (14) contains two utility discount rates. One (equal to δ) operates across a person's life, while the other (equal to 0) applies to the lifetime welfares of future generations.

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[2] At this point we merely assume without justification that expression (14) converges. Below we model the economy in such a way that it does converge.

Two extreme values of T define well known economic models:

(1) If $T = 0$, W_0 reduces to the formulation in Ramsey (1928).

(2) if $T = \infty$, W_0 represents the motivations of the infinitely lived household, familiar in contemporary macroeconomics.

A Single Dynasty in an Imperfect Economy

Suppose the rate of return on investment is r (> 0) and that households save a constant proportion s of their income ($0 < s < 1$). If $K(0)$ is the wealth person-*0* has inherited at birth ($t = 0$), her dynasty's wealth at $t \geq 0$ will be

$$K(t) = K(0)e^{srt}, \tag{15}$$

and consumption at t will be

$$C(t) = (1-s)rK(0)e^{srt}. \tag{16}$$

To have a meaningful problem, we assume $\eta s < 1$ (see below).

Previously we studied the consumption discount *rates* that are implied when intergenerational well-being is taken to be expression (1). We studied discount rates because they have been the objects of discussion in the economics of climate change. The items of real interest however are *discount factors*. They are the shadow prices of future consumption.

Using consumption at t=0 as *numeraire*, let $\alpha(t)$ denote the consumption discount factor for perturbations to $C(t)$. Equation (14) implies

$$\alpha(t) = [U'(C(t))/U'(C(0))]e^{-\delta(t-iT)}, \qquad \text{for } iT \leq t \leq (i+1)T, \text{ and } i = 0,1,... \tag{17}$$

As $C(t)$ satisfies equation (16), it is continuous in t. Therefore $\alpha(t)$ is discontinuous at iT (i = 1,2,...). That means consumption discount *rates* is undefined at each iT. If U satisfies equation (4) and $\eta > 1$, equations (16) and (17) tell us that

$$\alpha(t) = e^{-(\delta+\eta sr)t}e^{\delta iT}, \qquad \text{for } iT < t < (i+1)T, \text{ and } i = 0,1,... \tag{18}$$

Equation (18) says $\alpha(t)$ is a shrinking saw-tooth.

Staggered Dynasties in the Macroeconomy

We now construct a macroeconomy for a population of parallel dynasties. It is simplest to suppose the economy to be in steady state. So imagine that there is a continuum of staggered but otherwise identical dynasties. The population's age structure is uniformly distributed on $[0,T]$. Our analysis is conducted at $t = 0$.

Call the individual who is aged A (at $t = 0$), person-A; and call her dynasty, dynasty-A. By assumption $0 \leq A \leq T$. Let $C_A(t)$ denote consumption at t (≥ 0) by the member of dynasty-A and let $V_A(0)$ be intergenerational well-being of the dynasty-A. So,

$$V_A(0) = {}_0\int^{(T-A)}[U(C_A(t))e^{-\delta t}]dt + {}_{i=1}\Sigma^{\infty}[{}_{(iT-A)}\int^{((i+1)T-A)}\{U(C_A(t)e^{-\delta(t-(iT-A))}\}]dt. \quad (19)$$

Let $W(0)$ denote *aggregate* well-being at $t = 0$. For concreteness we assume that when it comes to public morality, citizens are Classical Utilitarianisms. From equation (1) we have

$$W(0) = {}_0\int^T [V_A(0)]\mathrm{d}A,$$

which on using equation (19) implies

$$W(0) = {}_0\int^T [{}_0\int^{(T-A)} \{U(C_A(t))\mathrm{e}^{-\delta t}\}\mathrm{d}t$$

$$+ {}_{i=1}\Sigma^{\infty} ({}_{(iT-A)}\int^{((i+1)T-A)} \{U(C_A(t)\mathrm{e}^{-\delta(t-(iT-A))}\})\mathrm{d}t]\mathrm{d}A. \qquad (20)$$

From equation (20) we are able to obtain consumption discount factors. Suppose $C_0(0)$ is *numeraire*. Let $\alpha_A(t)$ be the consumption discount factor for perturbations to $C_A(t)$. Then

$\alpha_A(t) = [U'(C_A(t))/U'(C(0))]e^{-\delta t}$, for $0 \leq t \leq (T\text{-}A)$,

and $\alpha_A(t) = [U'(C_A(t))/U'(C(0))]e^{-\delta[t-(iT-A)]}$, for $(iT\text{-}A) \leq t \leq ((i+1)T\text{-}A)$; and $i = 1,2...$ (21)

We can say more. Let $K_A(t)$ be the wealth of the member of dynasty-A at date t. Notice that each dynasty's wealth increases at the constant rate sr, as does its consumption rate. But because the economy is in steady state, $K_A(t)$ is related in a simple way to $K_0(0)$, the latter being the only boundary condition for the problem in hand.

Without loss of generality, suppose

$$K_A(0) = K_0(0)e^{rsA}, \qquad 0 \le A \le T. \tag{22}$$

Then

$$C_A(0) = (1-s)rK_0(0)e^{rsA}, \quad 0 \le A \le T. \tag{23}$$

From equations (22)-(23) we have

$$K_A(t) = K_0(0)\mathrm{e}^{rs(t+A)}, \qquad 0 \leq A \leq T, \tag{24}$$

and $$C_A(t) = (1\text{-}s)rK_0(0)\mathrm{e}^{rs(t+A)}, \qquad 0 \leq A \leq T. \tag{25}$$

Aggregate wealth at t is

$$K(t) = {}_0\int^T [K_A(t)]\mathrm{d}A, \tag{26}$$

and aggregate consumption at t is

$$C(t) = {}_0\int^T [C_A(t)]\mathrm{d}A$$

Calibrating the Representative Household

Now consider a welfare economist who misunderstands the economy and imagines that $C(t)$ mimics the optimum consumption rate of an infinitely lived household. The welfare economist also supposes that the representative household's utility function is iso-elastic (expression 4). Imagine next that the economist has obtained an estimate of η from micro data on people's choices under risk. He observes the aggregate savings rate to be s and knows that the economy has been growing at the constant rate sr. In his reckoning s is the optimum saving rate of the infinitely lived household. So he now uses equation (10) to conclude that the household discounts *all* future utilities at the constant rate δ, where

$$\delta = r(1-\eta s) > 0. \tag{27}$$

From equation (27) he then deduces that the consumption discount factor for evaluating perturbations to $C(t)$ is

$$\alpha(t) = [U'(C(t))/U'(C(0))]e^{-r(1-\eta s)t}, \qquad \text{for all } t \geq 0,$$

which reduces to

$$\alpha(t) = e^{-rt}. \tag{28}$$

On comparing equations (18) and (28), it is evident that the model of the infinitely lived household is wholly misleading.